O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4503
Random Signals and Noise
Spring 2004

Midterm Exam #1

Choose any four out of five.
Please specify below which four you choose to be graded.

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Name : ____________________________

Student ID: ____________________________

E-Mail Address: ____________________________________

Problem 1:
Given that two events $\overline{A}_1$ and $\overline{A}_2$ are statistically independent, show that A_1 is also statistically independent of A_2,
(i.e., given $P(\overline{A}_1 \cap \overline{A}_2) = P(\overline{A}_1)P(\overline{A}_2)$, prove $P(A_1 \cap A_2) = P(A_1)P(A_2)$.)

Problem 2:

Show if a random variable is symmetrical about $x = a$ (i.e., $f_X(x+a) = f_X(-x+a)$), then $E[x] = a$.

Problem 3:

In a game show, contestants choose one of three doors to determine what prize they win. History shows that the three doors, 1, 2, and 3, are chosen with probabilities 0.30, 0.45, and 0.25, respectively. It is also known that given door 1 is chosen, the probabilities of winning prizes of \$0, \$100, and \$1,000 are 0.10, 0.20, and 0.70. For door 2 the respective probabilities are 0.50, 0.35, and 0.15, and for door 3 they are 0.80, 0.15, and 0.05. If X is a random variable describing dollars won, and D indicates the door selected (values of D are $D_1 = 1$, $D_2 = 2$, and $D_3 = 3$), find a) $F_X(x|D = D_1)$, b) $f_X(x|D = D_2)$, and c) $f_X(x)$.

Problem 4:
A random variable X is uniformly distributed on $(0,6)$. If X is transformed to a new random variable via

$$Y = 2(X-3)^2 - 4,$$

find a) the density function $f_Y(y)$, b) $\overline{Y}$, and c) σ_Y^2.

Problem 5:
In a computer simulation, it is desired to transform numbers, that are values of a random variable uniformly distributed on (0,1), to numbers that are values of an Weibull distribution random variables, as defined by

$$F_X(x) = [1 - e^{-x^3/2}]u(x).$$

Find the required transformation.